Title: AJAE appendix for *Land Resilience and Tail Dependence Among Crop Yield Distributions*

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Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE)

Supplemental Materials

Item 1 *(demonstrating Proposition 1)*

Letting $x_1 \equiv y(w;\theta^a)$ and $x_2 \equiv y(w;\theta^b)$ we seek to show that the LTD condition applies, i.e., that $P[y(w;\theta^b) \le \hat{x}_2 \mid y(w;\theta^a) \le \hat{x}_1]$ is nonincreasing in $\hat{x}_1$ for all $\hat{x}_2$. Figure S1 depicts where $\hat{x}_2$ is arbitrarily chosen and $\hat{w}_2$ is the corresponding weather index value for $y(w;\theta^b)$ so that $P[y(w;\theta^b) \le \hat{x}_2] = F(\hat{w}_2)$. When $\hat{x}_1 = \hat{x}_1^*$ with corresponding weather quantile $\hat{w}_1^*$ then $P[y(w;\theta^a) \le \hat{x}_1^*] = P[w \le \hat{w}_1^*] = F(\hat{w}_1^*)$. In this case, whenever weather conditions are sufficiently bad to ensure that $y(w;\theta^a) \le \hat{x}_1^*$ then they will be sufficiently bad to ensure that $y(w;\theta^b) \le \hat{x}_2$ and so $P[y(w;\theta^b) \le \hat{x}_2 \mid y(w;\theta^a) \le \hat{x}_1^*] = 1$. However, when $\hat{x}_1 = \hat{x}_1^{**}$ then $y(w;\theta^b) \le \hat{x}_2$ is no longer guaranteed.

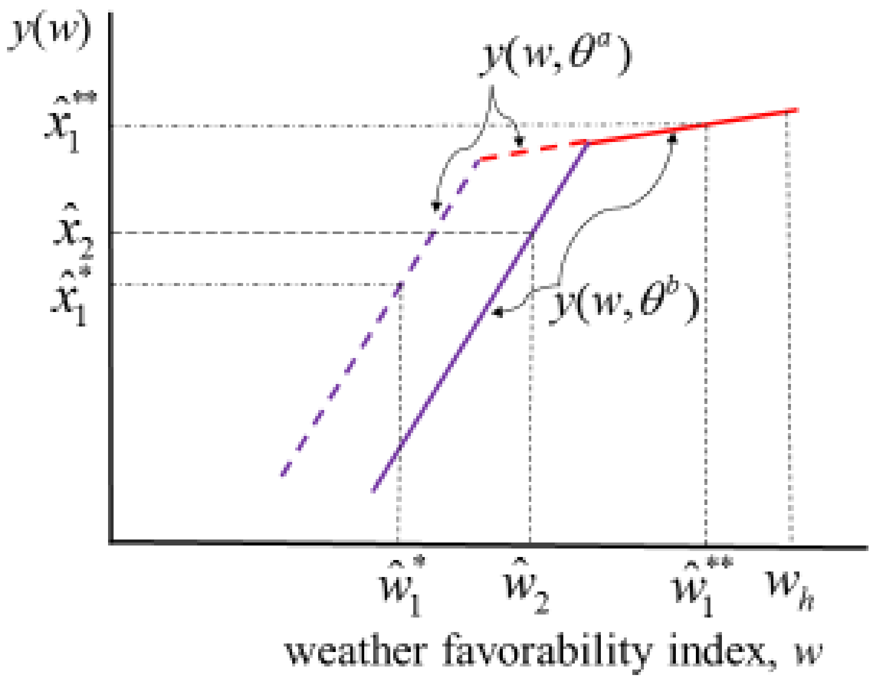
More generally,

(S1)
$$\begin{aligned} P[y(w;\theta^b) \le \hat{x}_2 \mid y(w;\theta^a) \le \hat{x}_1] &= \frac{P[\{y(w;\theta^b) \le \hat{x}_2\} \bigcap \{y(w;\theta^a) \le \hat{x}_1\}]}{P[y(w;\theta^a) \le \hat{x}_1]} \\ &= \frac{\min[F(\hat{w}_2), F(\hat{w}_1)]}{F(\hat{w}_1)} = \min\left[\frac{F(\hat{w}_2)}{F(\hat{w}_1)}, 1\right]. \end{aligned}$$

Now $\hat{x}_1$ is monotone increasing in $\hat{w}_1$ and $F(\hat{w}_2)/F(\hat{w}_1)$ is nonincreasing in $\hat{w}_1$ so that

$P[y(w;\theta^b) \le \hat{x}_2 \mid y(w;\theta^a) \le \hat{x}_1]$ is nonincreasing in $\hat{x}_1$. Notice that the inference above is symmetric in that it continues to apply if we interchange yield positions and consider instead $P[y(w;\theta^a) \le \hat{x}_2 \mid y(w;\theta^b) \le \hat{x}_1]$. We turn now to the RTI condition. Here we seek to show that $P[y(w;\theta^b) \ge \hat{x}_2 \mid y(w;\theta^a) \ge \hat{x}_1]$ is nondecreasing in $\hat{x}_1$ for all $\hat{x}_2$. The reasoning leading to (S1) above now supports $P[y(w;\theta^b) \ge \hat{x}_2 \mid y(w;\theta^a) \ge \hat{x}_1] = \min[\bar{F}(\hat{w}_2)/\bar{F}(\hat{w}_1),1]$. As $\hat{x}_1$ is monotone increasing in $\hat{w}_1$ and $\bar{F}(\hat{w}_2)/\bar{F}(\hat{w}_1)$ is nondecreasing in $\hat{w}_1$ it follows that $P[y(w;\theta^b) \ge \hat{x}_2 \mid y(w;\theta^a) \ge \hat{x}_1]$ is nondecreasing in $\hat{x}_1$.

Figure S1. Graphic of LTD Property for Land Yield Resilience Model



Item 2

The Bayesian estimation algorithm, or the Gibbs sampler, is a four-step procedure. All priors are chosen as conjugate when possible except those for the λ parameters, which are assumed to be normally distributed as usual. The four steps are:

Step 1: Draw Φ from the conditional posterior pdf of Φ:[1]

$$p(\Phi \mid \cdot) \sim N(D_\Phi d_\Phi, D_\Phi)$$

where $D_\Phi = \left[V_\Phi^{-1} + \sum_i \tilde{H}_i' \Sigma_i^{-1} \tilde{H}_i\right]^{-1}$, $d_\Phi = V_\Phi^{-1}\mu_\Phi + \sum_i \tilde{H}_i' \Sigma_i^{-1} \tilde{y}_i$, and $\Sigma_i = E(\varepsilon_i \varepsilon_i') = \sigma_\varepsilon^2 I_{T_i}$ is the variance-covariance matrix of county i, which is a diagonal matrix with the element σ_ε^2. Matrix I_{T_i} is the identity of size $T_i \times T_i$ while μ_Φ and V_Φ are, respectively, the mean and variance-covariance matrix of the prior normal distribution of Φ.

Step 2: Draw from the conditional posterior pdf of σ_ε^2:

$$p(\sigma_\varepsilon^2 \mid \cdot) \sim IG\left(0.5TN + a_\varepsilon,\ \left[b_\varepsilon^{-1} + 0.5\sum_{i=1}^{NT} (\tilde{y}_i - \tilde{H}_i \Phi)^2\right]^{-1}\right).$$

where $IG(\cdot)$ denotes the inverse gamma distribution and $T = \sum_i T_i$. We choose a conjugate prior $IG(a_\varepsilon, b_\varepsilon)$ with the shape and scale parameters of a_ε and b_ε.

Step 3: Draw $\eta \equiv (\eta_1, \ \ldots \ , \eta_N)'$ from the truncated normal distribution $N_{(-\sqrt{2/\pi}, \infty)}(\mu_W, \sigma_W^2)$ where $\mu_W = \left[\psi(\tilde{y}_i - \tilde{H}_i \Phi) - \sigma_\varepsilon^2 \sqrt{2/\pi}\right] / (\sigma_\varepsilon^2 + \psi^2)$ and $\sigma_W^2 = \sigma_\varepsilon^2 / (\sigma_\varepsilon^2 + \psi^2)$.

Step 4: Draw from the conditional posterior pdf of λ_j, $j \in \{1, 2, 3, 4\}$, the parameter in the *j*th element in $\tilde{X}_i$;

$$p(\lambda_j \mid \cdot) \propto e^{-0.5(\tilde{y}_i - \tilde{H}_i^{-j}\Phi^{-j} - \tilde{X}_i^j \phi^j)'(\tilde{y}_i^{-j} - \tilde{H}_i^{-j}\Phi^{-j} - \tilde{X}_i^j \phi^j)/\sigma_\varepsilon^2} e^{-0.5(\lambda_j - \mu_\lambda)^2/\sigma_\lambda^2},$$

where $\tilde{H}_i^{-j}\Phi^{-j}$ denotes the right-hand side variables and coefficients except the *j*th term in $\tilde{X}_i \phi$, $\tilde{X}_i^j \phi^j$. Expression $\tilde{X}_i^j \equiv \max[\lambda_j Q_i \pm \Upsilon_i, 0]$ involves the parameter λ_j and one of the two moisture variables P_i^L and P_i^R, which is denoted by Υ_i here.

The algorithm described above is coded in Matlab and ran 10,000 simulations. We discard the first 5,000 runs as burn-in and use the rest for inference. The initial values for parameters in Φ are all 0.5, those for λ_1 and λ_2 are 1, and $\mathrm{var}(\tilde{y}_i)$ for σ_ε^2.

Item 3

Estimation and Regression Results of Palmer's Z

Using the monthly Palmer's Z, precipitation (P) and temperature (T) from 1895-2015 for all climate divisions in South Dakota and Iowa, we estimate the following linear regression model (OLS),

$$Z_{i,t} = \beta_0 + \sum_{j=1}^{6} \beta_{Z,j} Z_{i,t-j} + \beta_1 \tilde{P}_{i,t} + \beta_2 \tilde{P}_{i,t}^2 + \beta_3 \tilde{P}_{i,t} \tilde{T}_{i,t} + \sum_M \beta_M \mathbf{1}_M \cdot T_{i,t} + \text{CDFE}$$

where $Z_{i,t}$ denotes the Palmer's Z of Climate division i in month t, Z_{t-j} is lagged Palmer's Z, $\tilde{P}_{i,t} = (P_{i,t} - \overline{P}) / std(P)$ is the standardized monthly precipitation, $\tilde{T}_{i,t} = (T_{i,t} - \overline{T}) / std(T)$ standardized monthly temperature, $\mathbf{1}_M$ represents month dummy, $\mathbf{1}_M \cdot T_{i,t}$ is the month-dummy interacted with monthly temperature, and CDFE are climate division fixed effects.

Note that monthly precipitation is total in the month, whereas monthly temperature is average for the month. The above regression has serious multi-collinearity when using higher-order polynomials for temperature and precipitation variables. So precipitation and temperature are standardized in order to reduce multi-collinearity and thus to allow the second order precipitation and temperature-precipitation interaction in the equation. Including monthly temperature and month-dummies separately would lead to high collinearity and dropping either reduces model-fit significantly. Hence, we include the interaction term between the two variables. The monthly dummies are included because evapotranspiration depends on average day length in a month

(Thornthwaite 1948). Overall, the effort has been to maximize fit and ensure stable coefficients that are then used for estimating Palmer-Z using futuristic weather projections of precipitation and temperature. It turns out that the model has a good fit with R^2 = 0.89. The estimation results are:

Table D-1. Estimation results of the Palmer's Z

Variable	Coeff. Estimates	Std. Error	*T* Value	Pr > |t|
Intercept	1.717	0.024	71.5	<0.0001
P	2.838	0.008	347.5	<0.0001
P^2	-0.150	0.003	-43.8	<0.0001
T*P	-0.337	0.008	-40.1	<0.0001
*Z*_lag1	0.172	0.002	77.5	<0.0001
*Z*_lag2	0.103	0.002	45.1	<0.0001
*Z*_lag3	0.058	0.002	25.2	<0.0001
*Z*_lag4	0.034	0.002	14.6	<0.0001
*Z*_lag5	0.034	0.002	15.0	<0.0001
*Z*_lag6	0.025	0.002	11.3	<0.0001
SD_DIV1	2.183	0.025	85.6	<0.0001
SD_DIV2	1.979	0.025	77.9	<0.0001
SD_DIV3	1.635	0.025	64.6	<0.0001
SD_DIV4	1.567	0.025	62.2	<0.0001
SD_DIV5	2.161	0.025	85.1	<0.0001
SD_DIV6	2.034	0.025	80.2	<0.0001

SD_DIV7	1.529	0.025	60.7	<0.0001
SD_DIV8	1.791	0.025	71.0	<0.0001
SD_DIV9	1.353	0.025	53.9	<0.0001
IA_DIV1	0.740	0.025	29.6	<0.0001
IA_DIV2	0.322	0.025	12.9	<0.0001
IA_DIV3	0.065	0.025	2.6	0.0086
IA_DIV4	0.563	0.025	22.6	<0.0001
IA_DIV5	0.252	0.025	10.1	<0.0001
IA_DIV6	-0.023	0.025	-0.9	0.3576
IA_DIV7	0.349	0.025	14.0	<0.0001
IA_DIV8	0.131	0.025	5.3	<0.0001
Jan_*T*	-0.000003	0.001	0.0	0.9973
Feb_*T*	-0.003	0.001	-3.7	0.0002
Mar_*T*	-0.053	0.001	-86.5	<0.0001
Apr_*T*	-0.066	0.0005	-134.7	<0.0001
May_*T*	-0.073	0.0004	-181.0	<0.0001
Jun_*T*	-0.073	0.0003	-217.4	<0.0001
Jul_*T*	-0.050	0.0003	-166.3	<0.0001
Aug_*T*	-0.050	0.0003	-163.1	<0.0001
Sep_*T*	-0.051	0.0004	-141.5	<0.0001
Oct_*T*	-0.040	0.0004	-90.9	<0.0001
Nov_*T*	-0.033	0.001	-56.1	<0.0001
N	26,136			

Adj. R^2	0.89

Reference

Thornthwaite, C.W. 1948. "An Approach toward a Rational Classification of Climate." *Geographical Review* 38(1): 55-94.

Footnote

1. Here $N(\mu, \Sigma)$ denotes a multivariate normal distribution with mean μ and variance-covariance matrix Σ.